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A new class of braneworld models displaying late-time phantom acceleration without resorting to a phantom fluid is presented. In this scenario expansion is fuelled by dark matter together with some effective dark energy capable of crossing the phantom divide. Unlike a previous proposal of this nature, in these models the effective phantom behaviour remains valid at all redshifts for some choices of the free parameters of the models. The construction is based on the generalised Chaplygin gas, and the cosmological history interpolates between a standard CDM-like behaviour at early times and a de Sitter-like behaviour at late times, so no future singularity is reached.

I. INTRODUCTION

Few things in Physics these days can be adjectivated as so intriguing as the reason for cosmological accelerated expansion on large scales. Observations of distant supernovae inform us of a departure from linearity in Hubble's law for which the most compelling explanation seems to be that the Universe is speeding up.

The simplest explanation is the presence of a cosmological constant or an exotic fluid making more than two thirds of the total matter/energy content of the Universe [1], but there is also the possibility the acceleration is a consequence of infrared modifications of gravity with respect to Einstein's theoretical framework like in the induced gravity braneworld scenario [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], which contains as a particular case the well-known DGP model [2]. At present we are far from being able to discard either interpretation, so in parallel to the observational developments required to answer the cosmic acceleration puzzle, considerable work is being done along the lines of theory also.

As the mathematical side of the problem is concerned, finding out about the kinematical/dynamical properties of a given model for the large scale Universe model starting off from a reconstruction of $H(z)$ is a well posed and clear-cut problem. In practice, however, the situation is tougher, as one only has at hand discrete datasets with disturbances (for various redshifts) from which one aims at extracting information applicable to all redshift values in a range of interest. Thus, in order to make progress it is vital to smooth out the quantities involved in the description.

Ideally, one would like to do this coarsening using a theoretically grounded description of the background geometry. Given that at present there is not a single model for dark energy which is completely satisfactory, the community keeps showing interest on new models which firstly do not depart substantially from the characteristics of other dark energy models in the market, secondly are likely to give a better fit to the data, and are thirdly are less problematic in what theoretical aspects are concerned.

Observational data do not seem incompatible with dark energy with phantom-like behaviour at $z = 0$ [17]. On the other hand, this radical behaviour may well be a property acquired by dark energy just recently, so models which do the so called phantom divide crossing keep gathering attention (for an overview of the problem see Ref. [18]). Quite a few realisations of this transition have been discussed in the literature. One can first distinguish general relativistic models, and, among, them there are models with an hybrid of quintessence and phantom dark energy called quintom [19, 20], models driven by a fluid called hessence models [21, 22] which originated after giving quintom models a turn of the screw, scalar field models with internal degrees of freedom [23], k-essence cosmologies [24], Chaplygin gas cosmologies [25] and models sourced by vector-like dark-energy [26]. Possibilities beyond general relativity are scalar tensor theories [27], $f(R)$ theories [28], Gauss-Bonnet gravity [29], and, finally, explanations which rely somehow on the assumption of the existence of extra dimensions (see for example [5, 6, 7, 8, 9, 10, 11, 12]).

A set of proposals to achieve that crossing are those made in the context of braneworld models and, in particular, those inspired in the LDGP model [5, 6]. Basically, those authors considered a 4-dimensional brane universe containing matter and a cosmological constant on the brane but no phantom fluid [5, 6]. The phantom-like behaviour was an effect due to the extra 5th dimension. A nice feature of the models is the effective screening of the cosmological

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constant, which results in the possibility that its value can be larger than the one observationally allowed in the LCDM case.

The LDGP cosmologies [5, 6] were generalised by replacing the cosmological constant with quiescence; i.e. a fluid with constant equation of state. These new models are referred to as QDGP cosmologies [7]. As a consequence of their more dynamical character, the phantom divide crossing is possible, unlike in the LDGP models¹. This rests on the possibility of describing the evolution as driven by an effective dark energy density which together with dust is considered to be the fuel of the geometry. Yet, these models can be criticised on the ground that the effective phantom picture always breaks down at some point as the effective dark energy density will cease to be positive at a sufficiently high redshifts (no matter the values of the free parameters of the model).

Here we put forward a possible remedy in the form of an alternative generalisation of the LDGP models, i.e. we present a class of models in which the phantom picture stays valid at all redshifts (given some conditions that can be met easily). This is achieved by taking the dark energy component on the brane to be a Chaplygin gas [30], so we can call these cosmologies Chaplygin DGP (hereafter CDGP) models.

Before proceeding, we recall that the Chaplygin gas by itself can behave as a phantom fluid [31] in the standard framework of Einstein gravity. However, this phantom fluid could induce a future singularity [32]. We will show that CDGP models also circumvent the future singularity issue that could arise in phantom Chaplygin gas models. Indeed, as we will show, in our case the brane is asymptotically de Sitter.

In order to show how our models do indeed display the features anticipated in this section, we will provide in the following sections analytical details on our construction, and for the sake of completeness, we will also present numerical examples to illustrate the effect of changing the values of the free parameters of the models.

II. THE MODEL

We consider a DGP₋ braneworld model² where the dark energy component on the brane is modelled by a generalised Chaplygin gas (GCG); i.e. $p_{\text{ch}} = -A/\rho_{\text{ch}}^\alpha$, and with an extra CDM component. In induced gravity braneworld models, which contain as a particular case the DGP scenario, if there is no interaction between the brane and the bulk the energy momentum tensor is conserved on the brane, so the usual conservation for the total matter/energy budget follows:

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (1)$$

but restricting it further so that conservation holds for the two components separately we have for the CDM bit

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (2)$$

which integrates to the usual $\rho_m = \rho_{m0}(1+z)^3$, whereas for the dark energy bit we have

$$\dot{\rho}_{\text{ch}} + 3H(\rho_{\text{ch}} + p_{\text{ch}}) = 0, \quad (3)$$

which results in

$$\rho_{\text{ch}} = \rho_{\text{ch}0} [A_s + (1 - A_s)(1+z)^{3(1+\alpha)}]^{-\frac{1}{1+\alpha}}. \quad (4)$$

The latter is the expression that supports the interest on Chaplygin cosmologies, as it reflects the fact that the energy density of such fluids interpolates between dust and a cosmological constant [30]. As customary the subscript 0 corresponds to the current value of a given quantity, and, in addition we define $A_s = A/(\rho_{\text{ch}0}^{1+\alpha})$.

Putting it all together, the modified Friedmann equation on the brane reads [3, 14, 15]

$$H/H_0 = \sqrt{\Omega_{r_c} + \Omega_m(1+z)^3 + \Omega_{\text{ch}} [A_s + (1 - A_s)(1+z)^{3(\alpha+1)}]^{-\frac{1}{1+\alpha}}} - \sqrt{\Omega_{r_c}}, \quad (5)$$

¹ Other generalisations of the LDGP model aiming at describing a mimicry of the phantom divide crossing in induced gravity braneworld models can be found in Refs. [10, 11, 12].

² The DGP cosmology has two different brane solutions, corresponding to the two possible ways of embedding the brane in the bulk [3, 16], the self-accelerating solution (or DGP₊ branch) and the DGP₋ branch. The latter is the one we are considering in the present work.

where

$$\Omega_m = \frac{\rho_{m0}}{3H_0^2}, \quad \Omega_{\text{ch}} = \frac{\rho_{\text{ch}0}}{3H_0^2}, \quad \Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}, \quad (6)$$

are the convenient usual dimensionless parameters.

Throughout the paper we will be assuming $0 < A_s < 1$ and $1 + \alpha > 0$, which imply that the braneworld goes over at low redshifts to a de Sitter cosmology (see Eq. (5)).

Similarly to what happened with vacuum energy in the precursor LDGP model [5, 6], we also have a screening effect here; the fractional amount of energy in the form of a Chaplygin gas in this CDGP model can be larger than in its general relativistic counterpart (which would be obtained on the $\Omega_{r_c} \rightarrow 0$ limit).

A consequence of Eq. (5), which leads to relevant insight on the model, is the constraint

$$\Omega_m + \Omega_{\text{ch}} = 1 + 2\sqrt{\Omega_{r_c}}. \quad (7)$$

It implies, on the first place that the region $\Omega_m + \Omega_{\text{ch}} < 1$ is unphysical (theoretical cuts of this sort are actually quite a source of complication when one tries to impose observational constraints [8, 9]). Secondly, although the brane is spatially flat, the constraint (7) can be interpreted in the sense there is a mimicry of closed FRW universes in the $(\Omega_m, \Omega_{\text{ch}})$ plane.

Now, a relevant question in this context is whether the brane super-inflates, and in order to answer to it the Raychaudhuri equation is of use:

$$\frac{\dot{H}}{H_0^2} = -\frac{3(1+z)^3 H}{2(H + H_0 \sqrt{\Omega_{r_c}})} \left\{ \Omega_m + \frac{(1 - A_s)\Omega_{\text{ch}}(1+z)^{3\alpha}}{[A_s + (1 - A_s)(1+z)^{3(1+\alpha)}]^{\frac{\alpha}{1+\alpha}}} \right\}. \quad (8)$$

At this point it is in order to stress that the Raychaudhuri equation in braneworld models is modified with respect to standard relativistic models, in such a way that the modification of both this equation and the Friedmann equation lead to a conservation equation for the matter/energy on the brane which in the absence of interaction between the bulk and the brane coincides with the conservation equation one would have in the relativistic situation.

Recently, an alternative route to the derivation of a cosmological model combining DGP gravity and a Chaplygin gas was taken by [33]. In this paper, the DGP-like modification of the Friedmann equation was combined with the relativistic form of the Raychaudhuri equation to derive a conservation equation for the possible fluids confined on the brane, the equation of state of them being the remaining necessary input. This procedure does not correspond to a construction derived from the DGP action (there is though the possibility an action might exist from which this derivation could be possible, but as this point has not been addressed in [33], such model can only be regarded as phenomenological at present).

In these models \dot{H} is never positive (cf. Eq. (8)), which means the brane never super-accelerates. Note, as well, that \dot{H} vanishes when $z \rightarrow -1$ while H is positive at this limit, thus reflecting their late-time de Sitter character. In contrast, the large z regime of the models is characterised by $2\dot{H} = -3H^2$ (this can be deduced by taking the $z \rightarrow \infty$ limit in Eqs. (5) and (8)). As the deceleration factor is well known to be $q = -(\dot{H}/H^2 + 1)$, and our brane cosmology transits between CDM ($q = 1/2$) and de Sitter ($q = -1$) limits there is always a certain redshift below which the brane accelerates (this redshift is signalled by $\dot{H} = -H^2$). In Fig. 1, the dimensionless deceleration parameter q is plotted against the redshift z for a fixed set of the parameters Ω_{r_c} and α . More precisely, we show the evolution of q in the left plot of Fig. 1 for a fixed amount of CDM and different values of A_s . This plot indicates that the late-time acceleration starts sooner the larger is the parameter A_s . On the other hand, we show the evolution of q for a fixed A_s and different amount of CDM in the right plot of Fig. 1. According to this graph, the larger is Ω_m , the later the beginning of the cosmic speed up.

By considering that the Universe is currently accelerating, i.e. $\dot{H} + H^2 > 0$, and imposing the constraint given in Eq. (7) it turns out that

$$2(\Omega_m + \Omega_{\text{ch}}) \leq 1 + 3A_s\Omega_{\text{ch}}. \quad (9)$$

This implies the additional constraint

$$\frac{1}{3\Omega_{\text{ch}}} \leq A_s \quad (10)$$

must be satisfied as well. Notice that the last inequality implies that the Chaplygin gas in this model cannot behave exactly as dust, i.e. $A_s \neq 0$.

In the next section we will discuss the main point of the paper, which is the possibility of having not just effective phantom behaviour, but also phantom divide crossing. The fact this behaviour is effective, i.e. it is achieved without having to resort to scalar fields with a wrong sign in the kinetic energy, allows bypassing the theoretical pathologies these models may display at the quantum level.

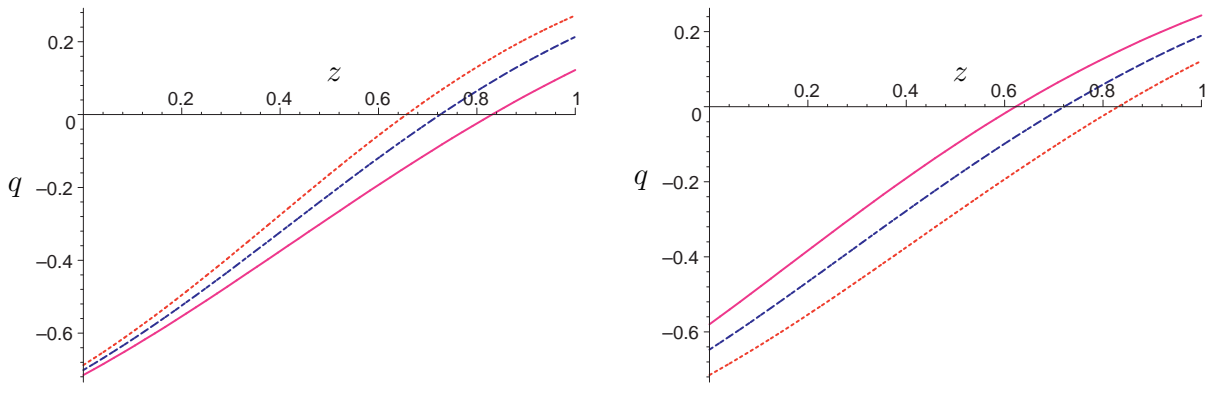


Figure 1: Representations of the dimensionless deceleration parameter q against the redshift z . The pair of parameters (Ω_{r_c}, α) has been fixed to $(0.01, 0.9)$. In the left plot $\Omega_m = 0.2$, and the pink solid line, blue dotted line and red dashed-dotted line correspond to $A_s = 0.991, 0.981, 0.971$, respectively. In the right plot $A_s = 0.991$, and the pink solid line, blue dotted line and red dashed-dotted line correspond to $\Omega_m = 0.3, 0.25, 0.21$, respectively.

III. CROSSING THE PHANTOM DIVIDE

As said in the introduction, as well as just above, the phantom behaviour in these CDGP cosmologies is an effective one, and it rests on the definition of an effective energy density and an effective equation of state parameter w_{eff} , so that:

$$3H^2 = \rho_m + \rho_{\text{eff}}, \quad (11)$$

$$\rho_{\text{eff}} = \rho_{\text{ch}} - 3\frac{H}{r_c}, \quad (12)$$

$$1 + w_{\text{eff}} = -\frac{\dot{\rho}_{\text{eff}}}{3H\rho_{\text{eff}}}. \quad (13)$$

It can be shown that the effective energy density does not vanish and is always positive if

$$4A_s(1 - A_s) > \left(\frac{4\Omega_{r_c}\Omega_m}{\Omega_{\text{ch}}^2}\right)^{1+\alpha}. \quad (14)$$

Specifically, the latter condition arises from imposing $\rho_{\text{eff}} = 0$ and the fulfilment of the Friedmann equation, which translates into a relation between ρ_m and ρ_{ch} , which in turn is equivalent to the vanishing of a quadratic polynomial on $(1+z)^{3(1+\alpha)}$. By requiring such equation has no roots, one just obtains expression (14). If this condition is met, the sign of $1 + w_{\text{eff}}$ is, therefore, completely determined by the sign of $\dot{\rho}_{\text{eff}}$ or equivalently by

$$\dot{\rho}_{\text{eff}} = \dot{\rho}_{\text{ch}} - 6\sqrt{\Omega_{r_c}}H_0\dot{H}. \quad (15)$$

A straightforward calculation shows

$$\dot{\rho}_{\text{ch}} = -\frac{9HH_0^2\Omega_{\text{ch}}(1 - A_s)(1+z)^{3(1+\alpha)}}{[A_s + (1 - A_s)(1+z)^{3(1+\alpha)}]^{\frac{\alpha}{1+\alpha}}}. \quad (16)$$

By combining Eqs. (4), (8) and (16), one gets

$$\dot{\rho}_{\text{eff}} = \frac{9HH_0^2(1+z)^3}{(H + H_0\sqrt{\Omega_{r_c}})} X \quad (17)$$

where

$$X = \Omega_m H_0 \sqrt{\Omega_{r_c}} - H(1 - A_s)\Omega_{\text{ch}}(1+z)^{3\alpha} [A_s + (1 - A_s)(1+z)^{3(1+\alpha)}]^{-\frac{\alpha}{1+\alpha}}. \quad (18)$$

At high redshifts ($z \gg 1$) the quantity $\dot{\rho}_{\text{eff}}$ is non-positive as

$$X \sim -H_0(1 - A_s)^{\frac{1}{1+\alpha}}\Omega_{\text{ch}}(1+z)^{\frac{3}{2}}\sqrt{\Omega_m + \Omega_{\text{ch}}(1 - A_s)^{\frac{1}{1+\alpha}}}. \quad (19)$$

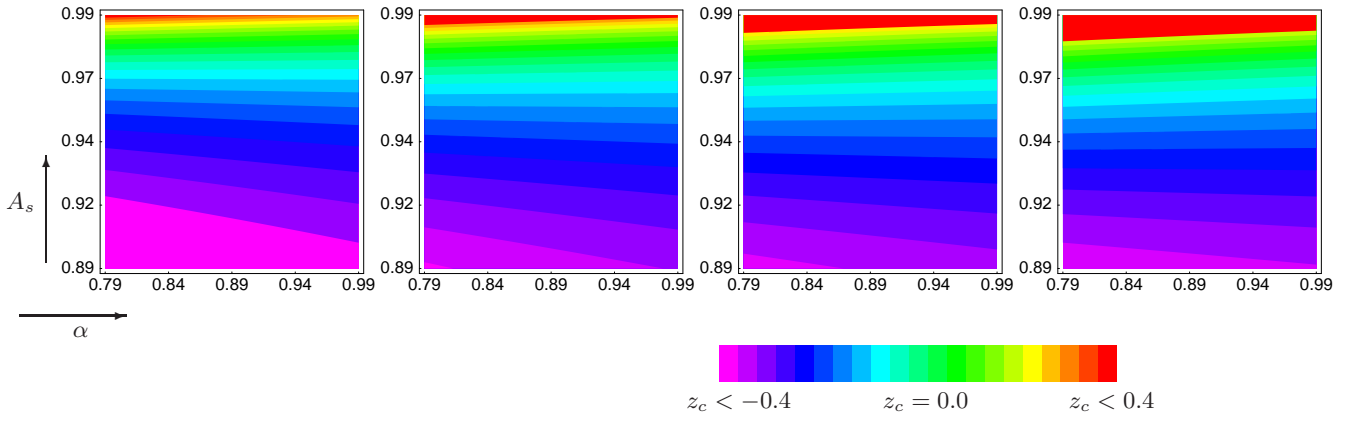


Figure 2: Contour plots of z_c as a function of α and A_s for $\Omega_{r_c} = 0.04$ and $\Omega_m = 0.20, 0.25, 0.30, 0.35$ (left to right).

Reversely, when $z \approx -1$ the quantity $\dot{\rho}_{\text{eff}}$ is non-negative, as one has

$$X \sim \Omega_m H_0 \sqrt{\Omega_{r_c}} \quad \text{for } \alpha > 0. \quad (20)$$

Thus, by noticing the opposite signs of X at the two redshift asymptotic regimes provided $\Omega_m \neq 0$, it follows that this condition is sufficient for $\dot{\rho}_{\text{eff}}$ to vanish at least once, that is, one is ensuring the realisation of a phantom divide crossing. In what follows we will show $\Omega_m \neq 0$ is not only sufficient but also necessary for the passage from the conventional to the phantom regime.

In the absence of CDM ($\Omega_m = 0$) the Raychaudhuri equation reads

$$2\dot{H} = -(1 + w_{\text{eff}})\rho_{\text{eff}}, \quad (21)$$

and since (in all cases) $\dot{H} \leq 0$ then $1 + w_{\text{eff}} \geq 0$ and the phantom divide crossing cannot occur. Just for the record, when a Chaplygin gas is advocated to account for both dust and a cosmological constant one speaks about a unified dark matter and dark energy scenario, whereas when the Chaplygin gas is considered in the simultaneous presence of additional dark matter the Chaplygin gas is regarded as a dark energy component. Our findings demonstrate that the Chaplygin gas cannot be the sole component of a DGP braneworld if it is to depict the transition from a non-phantom to a phantom-like regime. This seems to indicate that even though the original inspiration for the use of the Chaplygin gas was the unification of dark matter and dark energy, applications beyond that original motivation are indeed interesting.

Thus, summarising up to here, we can conclude that the phantom divide crossing is realisable in this scenario in the customary effective description, and we want to stress the fact this effective description is well defined at any redshift if condition (14) holds and α is positive.

It only remains to investigate whether with values of Ω_m close to observationally preferred ones, and values of Ω_{r_c} inspired by the closed LCDM “mimicry” ($\Omega_{r_c} \ll \Omega_m$) the crossing will occur at observationally accessible values (say $z \leq 1$ to be a little strict). This is a tricky task, as one would have on the one hand the difficulty of having to play with four parameters ($\Omega_m, \Omega_{r_c}, A_s, \alpha$) and on the other hand the manifest involvement of the pertinent equations. Thus, it does not seem possible to give taxative answers in an analytical way. For that reason, we have resorted to numerical means and, in particular we have done some contour plots to illustrate the effect of changing those parameters. Specifically, we select constant values of Ω_m and Ω_{r_c} and then draw contours of constant z_c (with z_c denoting the redshift value at which the crossing occurs). For the whole range of parameters considered the conditions that the universe is accelerating at present and that ρ_{eff} is positive are satisfied.

Our choice of range for α is arbitrary and our only reason for the particular values considered is that the examples stay close to the original Chaplygin gas ($\alpha = 1$). In contrast, our choice of range for A_s is based on some preliminary tests so that our joint choice of values of all four free parameters results in z_c values in the close past. Fig. 2 suggests a pattern according to which for a fixed value of Ω_{r_c} typically z_c grows with increasing Ω_m , whereas, similarly, in the view of Fig. 3 one would be inclined to say for a fixed value of Ω_m the typical z_c values grows with increasing Ω_{r_c} .

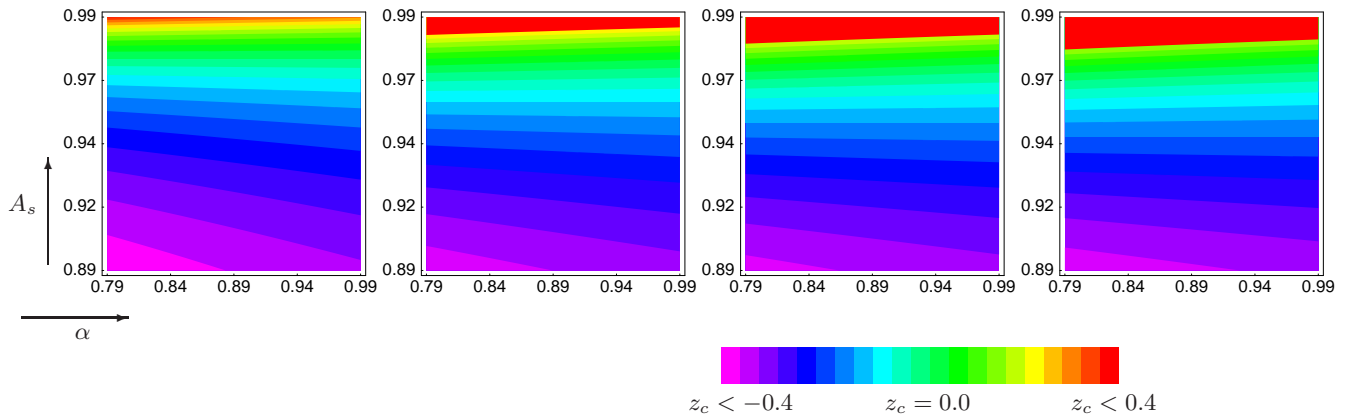


Figure 3: Contour plots of z_c as a function of α and A_s for $\Omega_{rc} = 0.04$ and $\Omega_m = 0.25$ and $\Omega_{rc} = 0.01, 0.04, 0.07, 0.10$ (left to right).

IV. CONCLUSIONS

Late-time cosmic acceleration is one of the major issues in Cosmology these days and we have addressed it from the perspective it is an extradimensional effect. We focus on a particular aspect of it, which has aroused a not disdainable activity: the possibility that the equation of state of dark energy (or its effective realisation in our case) has crossed the phantom barrier.

Here, we present an extra dimensions inspired model that is built on the DGP braneworld scenario [2], which pioneered the attempts of explaining cosmic acceleration initiated at recent times as an infrared modification of gravity [3, 4]. In broad terms the LDGP [5, 6] and QDGP [7] variants of the original DGP_ branch model (see footnote 2) can be viewed effectively as cosmological models filled with CDM and a phantom fluid. The QDGP model presents the advantage with respect to the LDGP model that the phantom divide crossing can be realised (so the effective fluid can be initially conventional as opposed to its latter phantom-like behaviour). However, both models have a drawback: the phantom picture breaks down at sufficiently high redshifts.

The content of the brane in the LDGP model and the QDGP models is respectively a cosmological constant and quiescence, adding in both cases a CDM component to form the whole matter/energy budget. Here we consider a model with a Chaplygin gas on top of CDM, and call it CDGP cosmological model. The advantages of this new scenario are that (i) the phantom effective picture does not cease to be valid at high redshifts and (ii) a mimicry of the phantom divide crossing is also possible, thus saving some of the problems associated to the LDGP and QDGP models. We also comment briefly on the consistency problems of another Chaplygin DGP model in the literature [33].

In the preceding sections, we have presented our model starting off from the Friedmann equation and we have analysed its main evolutionary features. Specifically, we have investigated the conditions for acceleration to occur, and we have concluded that it proceeds provided some restrictions are satisfied by the free parameters of the model. We have also addressed the question of super-acceleration, which we show to be unrealisable as in the LDGP and QDGP models³. Another aspect of the problem to which we devote considerable attention is the role of CDM, as it results crucial for the ability of the model to cross the phantom divide. Finally, we give a numerical account of the redshift values at which the crossing occurs, and with the support of some plots, we show there are values of the parameters for which the passage from the conventional to the phantom-like behaviour can occur at visible redshifts (see Fig. 2 and 3) while attaining at the same time acceleration at current times (see Fig. 1).

Acknowledgements

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³ See Ref. [34] for another variant of the DGP model where super-acceleration is possible and is driven by a bulk scalar field.

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